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Miscellaneous Exercise

Q1:- If A and B are symmetric matrices, prove that AB – BA is a skew symmetric matrix.

Notes for Q1:-

To prove AB – BA, a skew symmetric we need

$(AB - BA)' = -(AB - BA)$

For what we will use the following:-

1)Given A and B are symmetric matrices.

$\therefore A' = A$ and $B' = B$

2) $(A-B)' = A' - B'$

3) $(AB)' = B'A'$

Q1:- If A and B are symmetric matrices, prove that AB – BA is a skew symmetric matrix.

Given A and B are symmetric matrices.

$\therefore A' = A$ and $B' = B$ --- (1)

Now $(AB - BA)' = (AB)' - (BA)'$ **As** $(A-B)' = A' - B'$

$= B'A' - A'B'$ **As** $(AB)' = B'A'$

$= BA - AB$ using (1)

$= -(AB-BA)$

Since $(AB - BA)' = -(AB-BA)$

$\therefore$ AB – BA is a skew symmetric matrix.

Q2:- Show that the matrix $B'AB$ is symmetric or skew symmetric according as A is a symmetric or skew symmetric.

Here we have 2 cases:-

Case1:- To prove $B'AB$ is symmetric as A is symmetric.

i.e. $(B'AB)' = B'AB$

Case 2:- To prove $B'AB$ is skew symmetric as A is skew symmetric

i.e. $(B'AB)' = - B'AB$

To simplify

we will take AB as P

and we will use $A' = A$ and $\mathbf{(AB)' = B'A'}$ for case 1

and $A' = -A$ for case2.

We need to prove $B'AB$ is symmetric is A is symmetric or $B'AB$ is symmetric is A is skew symmetric .

Proving $B'AB$ is symmetric as A is symmetric

Let A be a symmetric matrix

Then $A' = A$ --(1)

Taking $(B'AB)'$

Let AB = P

Then $(B'AB)' = (B'P)' = (B')'\ P' = P'B$ **(as $(B')' = B$)**

Putting P =AB

$= (AB)'B = (B'A')B = B'AB$ **Using (1)**

So, as we have $(B'AB)' = B'AB$ **Thus. $B'AB$ is a symmetric matrix.**

Proving $B'AB$ is skew- symmetric is A is skew -symmetric

Let A be a skew symmetric matrix

Then $A' = -A$ --(2)

Taking $(B'AB)'$

Let AB = P

Then $(B'AB)' = (B'P)' = (B')' P' = P'B$ **(as $(B')' = B$)**

Putting P =AB

$= (AB)'B = (B'A')B = B'(-A)B$ **Using (2)**

$= -B'AB$

Thus. $B'AB$ is a skew symmetric matrix.

Hence Matrix $B'AB$ is a symmetric or skew symmetric matrix according as A is a symmetric or skew symmetric.

Q3:- Find the values of x, y, z if the Matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies the equation $A' A = I$

Given, $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$

$$A' = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now A'A = I

Putting Values

$$\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0+x^2+x^2 & 0+xy-xy & 0-xz+xz \\ 0+xy-xy & 4y^2+y^2+y^2 & 2zy-zy-zy \\ 0-xz+xz & 2zy-zy-zy & z^2+z^2+z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the matrices are equal, therefore the corresponding elements will be equal too.

$2x^2 = 1$ $6y^2 = 1$ $3z^2 = 1$

$x^2 = \frac{1}{2}$ $y^2 = \frac{1}{6}$ $z^2 = \frac{1}{3}$

$x = \pm\sqrt{\frac{1}{2}}$ $y = \pm\sqrt{\frac{1}{6}}$ $z = \pm\sqrt{\frac{1}{3}}$

Thus $x = \pm\sqrt{\frac{1}{2}}, y = \pm\sqrt{\frac{1}{6}}, z = \pm\sqrt{\frac{1}{3}}$

Q4:- For what values of x, $[1\ 2\ 1]\begin{bmatrix}1 & 2 & 0\\ 2 & 0 & 1\\ 1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix} = 0$

A **B** **C**

$$\textbf{AB} = [1\ 2\ 1]\begin{bmatrix}1 & 2 & 0\\ 2 & 0 & 1\\ 1 & 0 & 2\end{bmatrix} = [1(1)+2(2)+1(1) \quad 1(2)+2(0)+1(0) \quad 1(0)+2(1)+1(2)]$$

Note:- We will get AB first and then we will multiply that with C. Note:- A is a 1 × 3 matrix and B is a 3 × 3 matrix so AB will be 1× 3 and AB(1 ×3) × C (3×1) will be 1 × 1

Now ATQ

$$= [6 \quad 2 \quad 4]\begin{bmatrix}0\\ 2\\ x\end{bmatrix} = 0$$

$$= [6(0) + 2(2) + 4(x)] = 0$$

$4 + 4x = 0$

$x = -1$

HW:-

Q6:- (It is similar to Q4.)

Q5:- We will find A^2 and then put the values in the given equation. Note. I will be in the form of 2*2 Identity matrix and then it will be multiplied with 7.

Q7:- A manufacturer produces three products x,y,z which he sells in two markets.

Annual sales are indicated below:

Market	Products		
I	10,000	2,000	18,000
II	6,000	20,000	8,000

(a)If unit sale prices of x,y and z are ₹2.50, ₹1.50 and ₹1.00, respectively, and

(b)If unit costs of x,y and z are ₹2.00 and ₹1.00, ₹ 50 paise respectively.

Find the gross profit.

(a) Let the sale of products x, y and z per market be denoted by Matrix A

$$\begin{matrix} & x & y & z & \\ & \begin{bmatrix} 10,000 & 2,000 & 18,000 \\ 6,000 & 20,000 & 8,000 \end{bmatrix} & & & \begin{matrix} \textbf{Market I} \\ \textbf{Market II} \end{matrix} \end{matrix}$$

Let the unit sale price of products x, y and z be denoted by Matrix B

$$\text{Let } B = \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix} \begin{matrix} \mathbf{x} \\ \mathbf{y} \\ \mathbf{z} \end{matrix}$$

Now Total Revenue = Total sales × Unit Sales price = AB

$$\begin{bmatrix} 10,000 & 2,000 & 18,000 \\ 6,000 & 20,000 & 8,000 \end{bmatrix} \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix} = \begin{bmatrix} 10,000(2.50 + 2000\ (1.50) + 18000(1) \\ 6,000(2.50) + 20,000(1.50) + 8000(1) \end{bmatrix}$$

$$= \begin{bmatrix} 46,000 \\ 53,000 \end{bmatrix} \begin{matrix} \textbf{Market I} \\ \textbf{Market II} \end{matrix}$$

Hence, total revenue of market I= Rs. 46,000 and for Market II = Rs. 53,000

(b) Let the Sale of products x, y and z per market be denoted by Matrix A

$$\begin{array}{c} \quad \mathbf{x} \qquad\qquad \mathbf{y} \qquad\qquad \mathbf{z} \\ \begin{bmatrix} 10{,}000 & 2{,}000 & 18{,}000 \\ 6{,}000 & 20{,}000 & 8{,}000 \end{bmatrix} \begin{array}{l} \textbf{Market I} \\ \textbf{Market II} \end{array} \end{array}$$

Let the unit cost price of products x, y and z be denoted by Matrix B

$$\text{Let } B = \begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix} \begin{array}{l} \mathbf{x} \\ \mathbf{y} \\ \mathbf{z} \end{array}$$

Now Total Cost = Total sales × Unit cost price = AB

$$\begin{bmatrix} 10{,}000 & 2{,}000 & 18{,}000 \\ 6{,}000 & 20{,}000 & 8{,}000 \end{bmatrix} \begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix} = \begin{bmatrix} 10{,}000(2.00 + 2000\ (1.00) + 18000(0.50) \\ 6{,}000(2.00) + 20{,}000(1.00) + 8000(0.50) \end{bmatrix}$$

$$= \begin{bmatrix} 31,000 \\ 36,000 \end{bmatrix} \begin{matrix} \textbf{Market I} \\ \textbf{Market II} \end{matrix}$$

Hence, total cost of market I= Rs. 31,000 and for Market II = Rs. 36,000

Now, Profit = Revenue –Cost

$$= \begin{bmatrix} 46,000 \\ 53,000 \end{bmatrix} - \begin{bmatrix} 31,000 \\ 36,000 \end{bmatrix} = \begin{bmatrix} 15,000 \\ 17,000 \end{bmatrix} \begin{matrix} \textbf{Market I} \\ \textbf{Market II} \end{matrix}$$

Hence, total profit of market I= Rs.15,000 and for Market II = Rs. 17,000

Q8 Find the matrix X so that $\underset{2\times 2}{X}\underset{2\times 3}{\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}}\underset{2\times 3}{\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}}$

The matrix X should be of order of 2 × 2 only as No. of column of first matrix should match with the no. of rows of second matrix to get the resultant matrix of 2 × 3

$$\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix} = \begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$$

$$\begin{bmatrix}a+4b & 2a+5b & 3a+6b\\ c+4d & 2c+5d & 3c+6d\end{bmatrix} = \begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$$

Since the matrices are equal. So, the corresponding elements will be equal too.

a+4b =-7 ---(1)

2a+5b =−8 ---(2)

Solving (1) and (2) we get a= 1and b = -2

c+4d =2 ---(3)

2c+5d =4 ---(4)

Solving (3) and (4) we get c= 2 and d = 0

Hence the matrix X is $\begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

Q9:- If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$ then

A. $1 + \alpha^2 + \beta\gamma = 0$
B. $1 - \alpha^2 + \beta\gamma = 0$
C. $1 - \alpha^2 - \beta\gamma = 0$
D. $1 + \alpha^2 - \beta\gamma = 0$

$A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$

Given that $A^2 = I$
$A.A = I$

$$\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \alpha^2 + \beta\gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Since the matrices are equal, corresponding element are also equal

$\therefore \alpha^2 + \beta\gamma = 1$

$0 = 1 - \alpha^2 - \beta\gamma$

$1 - \alpha^2 - \beta\gamma = 0$

Hence C is the correct Answer.

Q10:- If the Matrix A is both symmetric and skew symmetric, then

A. A is a diagonal matrix
B. A is a zero matrix
C. A is a square matrix
D. None of these

DIAGONAL MATRICE: Matrix with all Non diagonal elements ZERO

Example: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

NULL MATRICE : Matrix with all elements ZERO

Example: $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

SQUARE MATRICE: Matrix with No of columns = No. of rows

Example: $\begin{bmatrix} 9 & -6 & 3 \\ 2 & -8 & 5 \\ 2 & -4 & 6 \end{bmatrix}$

Since A is both symmetric and skew-symmetric matrix,

$\therefore A' = A$

And $A' = -A$

Comparing both equations

A=-A

2A =0

A=0

$\therefore$ A is a zero matrix.

$\therefore$ B is the correct Answer.

Q11:-

If A is a Square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to

A. A   B. I-A   C. I   D. 3A

Given that $A^2 = A$

Using $(a + b)^3 = (a)^3 + (b)^3 + (3a^2b) + (3ab^2)$

$(I + A)^3 = (I)^3 + (A)^3 + (3I^2A) + (3IA^2)$

$(I + A)^3 = I + A^2 A + 3A + 3A^2$

$= I + A.A + 3A + 3A$

$= I + A^2 + 6A$

$= I + A + 6A = I + 7A$

Now $(I + A)^3 - 7A = I + 7A - 7A = I$. Hence the correct Answer is I